2020

MATHEMATICS — HONOURS

Seventh Paper

(Module - XIV)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

[Probability]

(Marks : 30)

Answer ***any one*** question.

1. (a) State the axioms of probability and give frequency interpretation of the axioms. What is meant by probability space?

(b) For n events A_1, A_2, ..., A_n in a probability space show that

$$P(A_1 A_2 \ldots A_n) \geq P(A_1) + P(A_2) + \ldots + P(A_n) - (n-1).$$

Hence deduce $P(A_1 + A_2 + \ldots + A_n) \leq P(A_1) + P(A_2) + \ldots + P(A_n)$.

(c) From an urn containing 3 white and 5 black balls, 4 balls are transferred into an empty urn. From this urn a ball is drawn and is found to be white. What is the probability that out of four balls transferred 3 are white and 1 is black? (4+2+4)+(6+4)+10

2. (a) If m objects are distributed at random among a men and b women, then show that the probability that men will get an odd number of objects is $\dfrac{(a+b)^m - (b-a)^m}{2(a+b)^m}$.

(b) In a Bernoullian sequence of n trials with constant probability of success p, find the most probable number of successes.

(c) When is a random variable said to be continuous? If a random variable X has standard normal distribution, find the probability density function of Y, where $Y = \dfrac{X^2}{2}$. 10+10+(2+8)

3. (a) Find the moment generating function of uniform distribution in $(-a, a)$, $a > 0$. Hence find moments of order k about the origin, where k is a positive integer. Also find the central moment of order 6.

(b) If X is a Poisson variate with parameter μ, show that $P(X \leq n) = \frac{1}{\lfloor n} \int_{\mu}^{\infty} e^{-x} x^n dx$ where n is any positive integer.

(c) When are two random variables said to be independent? If p and q be independent random variables each uniformly distributed over the interval (–1, 1), find the probability that the equation $x^2 + 2px + q = 0$ has real roots. (4+4+2)+10+(2+8)

4. (a) State the limit theorem for characteristic functions. With the help of this theorem derive Poisson distribution as a limit of the binomial distribution.

[Hint : The characteristic function for a binomial (n, p) variate is $\left(pe^{it} + 1 - p\right)^n$ and the characteristic function for a Poisson (μ) distribution is $e^{\mu\left(e^{it}-1\right)}$]

(b) Let $U = X + aY$, $V = X + \frac{\sigma_x}{\sigma_y} Y$, where a is a constant and σ_x, σ_y are the standard deviations of the random variables X, Y where X, Y are positively correlated. If $\rho(U, V) = 0$ then show that $a = -\frac{\sigma_x}{\sigma_y}$, where $\rho(U, V)$ is the correlation coefficient of U and V.

(c) Let $g : \mathbb{R} \to \mathbb{R}$ be a non-decreasing function such that $g(x) > 0$ for all $x \in \mathbb{R}$ and $E(X) = m$, where X is a random variable. If $E\{g(|X - m|)\}$ exists, then for any $\in > 0$

$$P(|X - m| \geqslant \in) \leq \frac{E\{g(|X - m|)\}}{g(\in)}$$

(4+6)+10+10

Group - B

[Statistics]

(Marks : 20)

Answer ***any one*** question.

5. (a) Distinguish between 'distribution of a population' and 'distribution of a sample'. Explain the statement : "Distribution of the sample is the statistical image of the distribution of the population."

(b) For a normal (μ, σ) population, show that the statistic $\frac{nS^2}{\sigma^2}$ is χ^2-distributed with $(n - 1)$ degrees of freedom where n, S^2, σ^2 are sample size, sample variance and population variance respectively; μ is the population mean.

(c) A bivariate sample of size 11 gave the results $\bar{x}=7, S_x=2, \bar{y}=9, S_y=4$ and $r = 0.5$. It was later found that one pair of the sample values ($x = 7$, $y = 9$) was inaccurate and was rejected. How would the original value of r be affected by the rejection? (The symbols have their usual meaning)

(d) The random variable X is normally distributed with mean 68 cms and s.d. 2.5 cms. What should be the size of the sample whose mean shall not differ from the population mean by more than 1 cm with probability 0.95?

[Given that the area under standard normal curve to the right of the ordinate at 1.96 is 0.025]

(1+1+2)+7+5+4

6. (a) Find the maximum likelihood estimate for the parameter p of a binomial (2020, p) population on the basis of a sample drawn from the population. Is this estimate consistent?

(b) Find a confidence interval for the parameter m of a normal (m, σ) population with confidence coefficient $1 - \epsilon$ $(0 < \epsilon < 1)$ on the basis of a sample drawn from the population, where σ is known.

(c) Explain : (i) Simple hypothesis, (ii) Composite hypothesis, (iii) Critical region, (iv) Type-II error, (v) Power of a test.

(d) Design a decision rule to test the hypothesis that a coin is fair, if a sample of 64 tosses of the coin is taken and if a level of significance of 0.05 is used.

Given that $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^{1\cdot96} e^{-x^2/2}\, dx = 0.4750$

(e) Find by the method of likelihood ratio testing, a test of $H_o : \sigma = \sigma_o$ for a normal (m, σ) population assuming that m is known.

(3+1)+3+5+4+4